Designing for Six-Sigma Quality with Robust Optimization Using CAE

Andreas Vlahinos
Advanced Engineering Solutions, LLC

Subhash G. Kelkar
Ford Motor Company

ABSTRACT

Although great advances have been made over the last two decades in the automotive structural design process, tradition and experience guide many design choices even today. The need for innovative tools is stronger now more than ever before as the design engineer is confronted with more complex, often contradictory design requirements such as cost, weight, performance, safety, time to market, life cycle, aesthetics, environmental impact, changes in the industry’s business models, etc.

The ever-increasing use of optimization tools in engineering design generates solutions that are very close to the limits of the design constraints, hardly allowing for tolerances to compensate for uncontrollable factors such as manufacturing imperfections. Optimum designs developed without consideration of uncertainty can lead to non-robust designs. Reliability-Based Design Optimization (RBDO) methodologies not only provide improved designs but also a confidence range for simulation-based optimum designs.

In this research effort, a six-sigma robust design formulation is presented along with an example that demonstrates the advantage of robust versus deterministic optimization.

INTRODUCTION

The need for innovative tools is apparent now more than ever as more complex design requirements are surfacing such as cost, performance, safety, quality, time to market, short life cycle, environmental impacts, wow aesthetics (creating a passion for the product: “I’ve got to have it”) and major changes in industries’ business models. Moreover, the automotive industry’s cycle development time from concept to production is being compressed significantly. Some of the changes in the automotive industry’s business model include: vehicle designs are tailored to focused markets; vehicles are being manufactured more on a global scale; and vehicles are designed increasingly through multiple engineering sites around the world.

Quality issues are addressed early in the design cycle with robust design as a methodology. The goal of robust design is to deliver customer expectations at affordable cost regardless of customer usage, degradation over product life and variation in manufacturing, suppliers, distribution, delivery and installation. Since randomness and scatter is a part of reality everywhere, probabilistic design techniques are necessary to engineer quality into designs. Traditional deterministic approaches account for uncertainties through the use of empirical safety factors. The safety factors are derived based on past experience; they do not guarantee satisfactory performance and do not provide sufficient information to achieve optimal use of available resources. The probabilistic design process has not been widely used because it has been intimidating and tedious due to its complexity. In recent years, FEA codes have introduced integrated probabilistic systems (e.g. ANSYS PDS) that make probabilistic analysis setup simple if the control and the noise parameters are identifiable [Ref. 1]. Control parameters are those factors that the designer can control, such as geometric design variables, material selection, design configurations and manufacturing process settings [Ref. 2]. Noise parameters on the other hand are factors that are beyond the control of the designer, such as material property variability, manufacturing process limitations, environment temperature, humidity, component degradation with time etc.

This paper describes a technique to perform probabilistic analysis, reliability based optimization and robust optimization.
FORMULATIONS FOR DETERMINISTIC, RELIABILITY AND ROBUST OPTIMIZATION

Deterministic Optimization

The ever-increasing use of optimization tools in engineering designs generates designs that are on the design constraint limits leaving very little or no room for tolerances in modeling uncertainties and manufacturing imperfections [Ref. 3]. Optimum designs developed without consideration of uncertainty could possibly lead to unreliable designs. In addition to the active constraint problems, optimum designs may be sensitive to design parameters such that small changes in the design variables may lead to a significant loss of performance. A possible formulation of deterministic optimization from automotive crashworthiness (with fictitious numbers) can be:

Minimize \( \text{Weight} \),
subject to: \( B\)-Pillar Velocity \( \leq \) 10 mm/s
Abdomen Load \( \leq \) 1.0 KN
Rib Displacement \( \leq \) 30 mm
Symphysis Load \( \leq \) 5.0 KN

Thickness Design Variables:
\( t_{\text{min}} \leq t \leq t_{\text{max}} \)

Reliability Index Approach

For reliability based design, a performance function can be defined as \( G = R - S \) where \( R \) and \( S \) are statistically independent and normally distributed random variables of the resistance and load measurements of the structure. Typically, \( R \) can be the yield stress and \( S \) the maximum Von Mises stress. The \( G \) function is also called limit state function or failure function. The curve \( G = 0 \) divides the design space into two regions, the safe region when \( G > 0 \) and unsafe region when \( G < 0 \). Because we consider \( R \) and \( S \) to have variation, \( G \) will also exhibit variation. The ratio \( \beta \) of the mean value of the \( G \) function (\( \mu_G \)) and the standard deviation of the \( G \) function (\( \sigma_G \)) is defined as safety index or reliability index. If \( \Phi \) is the cumulative distribution function and \( G \) has a normal distribution, then:

\[ \beta = -\Phi(1 - \text{Reliability}) = \frac{\mu_G}{\sigma_G} \]

A possible formulation can be:

Maximize \( \beta \),
subject to: Weight \( < \) Target-Weight
\( D V_{\text{min}} < \) \( D V \) \( < \) \( D V_{\text{max}} \)

Another formulation using the reliability index can be:

Minimize \( \text{Weight} \),
subject to: \( \beta > \) Target \( \beta \)
\( D V_{\text{min}} < \) \( D V \) \( < \) \( D V_{\text{max}} \)

A typical target value \( \beta \) is 3, which corresponds to a probability of failure of 0.00135. However, it has been observed that the Reliability Index Approach exhibits very slow convergence or even divergence for some problems.

Reliability Optimization Approach

Significant research effort has been devoted to making reliability based structural design optimization practical [Ref. 4, 5, 6]. Reliability based optimization requires two major steps. First, we need to identify the random variables and qualify the causes of variation. Typically the distribution type and the necessary values that describe the distribution function of the random variables (i.e. mean and standard deviation for a normal distribution) need to be found. Second, we need to select the desired reliability level (say, 90%) and reformulate the deterministic constraints as probabilistic constraints. For example a 90% reliability goal may be expressed as: the probability of failure \( P_f \) of the Abdomen Force to be greater than 1 KN must be less than 10%. A possible formulation of reliability-based optimization with a reliability goal of 95% from automotive crashworthiness (with fictitious numbers) can be:

Minimize \( \text{Weight} \),
subject to:
\( P_{\text{failure}} [\text{B-Pillar Velocity} > 10 \text{ mm/s}] \leq 5\% \)
\( P_{\text{failure}} [\text{Abdomen Load} > 1.0 \text{ KN}] \leq 5\% \)
\( P_{\text{failure}} [\text{Rib Displacement} > 30 \text{ mm}] \leq 5\% \)
\( P_{\text{failure}} [\text{Symphysis Load} > 5.0 \text{ KN}] \leq 5\% \)

Design Variables:
Mean values of various design parameters,
\( \mu_{t_{\text{min}}} \leq \mu_t \leq \mu_{t_{\text{max}}} \)
Random Variables with known or assumed variation:
Thickenes
Yield Stress
Barrier Height
Impact Position
The reliability based optimization approach accounts for variation and generates designs that meet a given level of reliability and usually move optimum solutions away from the constraints.

**Robust Optimization Approach**

The robust design optimization approach not only shifts the performance mean to the target value but also reduces the product’s performance variability, achieving **Six-sigma level** robustness on the key product performance characteristics with respect to the quantified variation [Ref. 7, 8]. A possible formulation of a 6-sigma level robust design optimization approach from automotive crashworthiness (with fictitious numbers) can be:

Minimize Weight,

subject to:

\[
\mu \left[ B\text{-Pillar Velocity} \right] + 6 \sigma \left[ B\text{-Pillar Velocity} \right] \leq 10 \text{ mm/s}
\]

\[
\mu \left[ Abdomen Load \right] + 6 \sigma \left[ Abdomen Load \right] \leq 1.0 \text{ KN}
\]

\[
\mu \left[ Rib Displacement \right] + 6 \sigma \left[ Rib Displacement \right] \leq 30 \text{ mm}
\]

\[
\mu \left[ Symphysis Load \right] + 6 \sigma \left[ Symphysis Load \right] \leq 5.0 \text{ KN}
\]

Design Variables: Mean values or standard deviation of design parameters,

\[
\mu_{t_{\text{min}}} \leq \mu_t \leq \mu_{t_{\text{max}}}
\]

\[
\sigma_{t_{\text{min}}} \leq \sigma_t \leq \sigma_{t_{\text{max}}}
\]

Random Variables with known or assumed variation:

- Thickness
- Yield Stress
- Barrier Height
- Impact Position

For any combination of the input parameters (t and P) the solution to the parametric model can compute the maximum Von Mises stress \( \sigma_{eq} \). We define a performance function \( G \) as the difference between yield stress and the maximum computed Von Mises stress or

\[
G = \sigma_y - \sigma_{eq}
\]

If \( G \) remains positive at all time, we have a safe design.

The performance function \( G \) is considered the output variable and is a function of the input variables \( t \) and \( P \).

The data flow for the parametric FE model is shown in the blue box in Figure 3. For a given geometry and a set of values of the input parameters, the deterministic parametric FE model can predict the value of the performance function \( G \).

**The Parametric Deterministic FEA Model**

In this research effort, the probabilistic modeling of manufacturing (thickness) and loading variations for the radiator support of a SUV was considered. A parametric finite element model of the radiator support was developed considering the thickness \( t \) and the load \( P \) as parameters. It was assumed that the material is isotropic, linear elastic and the behavior is within small deflection linear theory limits. The Modulus of Elasticity, \( E \) is assumed to be 200000 MPa and Poisson’s ratio, \( \nu = 0.25 \). Several load cases were evaluated and the torsional load case was found to control the maximum Von Mises stress. The distribution of the magnitude of the displacement vector for this load case is shown in Figure 1. The corresponding Von Mises stress distribution is shown in Figure 2.
The Probabilistic FEA Model

Uncertainty in the input parameters of the FEA model can be introduced by assuming certain randomness in the input parameters. In this study, it was assumed that a Gaussian distribution with mean value $\mu$ and standard variation $\sigma$ characterizes the variation in thickness $t$. The mean value $\mu$ was considered as a **controllable** parameter and it was declared an **optimization design variable**. The standard variation $\sigma$ was considered as 3% of the mean value $\mu$. The applied load $P$ was considered as uncontrollable or a noise parameter that exhibits a lognormal distribution with a mean value $\mu = 44$ N and standard variation $\sigma = 8.8$ N. Figure 4 shows the Probability Distribution of the input variable $P$.

Monte Carlo and the Central Composite Design response surface sampling techniques were implemented in determining the response distribution of the output variable $G$ for various values of the mean thickness value $\mu$. Figure 3 shows the data flow of the probabilistic model. A sampling method is selected that combines both noise and control random variables to produce a set of deterministic runs, using the parametric FE model. Post processing of these runs determines the probability distribution of the performance function $G$.

Figure 5 shows the Probability Distribution for three different values of mean thickness $\mu = 0.85, 1.0$ and $2.0$. One may observe that for $\mu = 2.0$, the entire distribution of the performance function $G$ remains on the positive side indicating that for $\mu = 2.0$ the maximum Von Mises stress does not exceed the yield shear stress. In this case the mean value of the performance function $\mu = 163.47$ MPa, the standard deviation $\sigma = 7.56$ MPa and the probability that the performance function $G$ is less than zero is 0%, $P[G<0] = 0%$.

The top part of Figure 5 shows the Probability Distribution of the input variable $t$ with mean value $\mu = 1.10$ mm and standard variation $\sigma = 0.033$ mm. The lower part of Figure 5 shows the Probability Distribution of the input variable $t$ with mean value $\mu = 0.88$ mm and standard variation $\sigma = 0.0264$ mm.

For $\mu = 1.0$, one may observe that part of the distribution of the performance function $G$ remains on the positive side indicating that for $\mu = 1.0$ the maximum Von Mises stress some times exceeds the yield shear stress. The...
area to the left of the zero (red line) indicates the probability of failure. In this case, the mean value of the performance function $\mu_G = 64.676$ MPa, the standard deviation $\sigma_G = 27.90$ MPa, and the probability that the performance function $G$ is less than zero is 2.3%, $P[G<0] = 2.3\%$.

Similarly for $\mu_t = 0.88$, a smaller part of the distribution of the performance function $G$ remains on the positive side indicating that for $\mu_t = 0.88$ the maximum Von Mises stress sometimes exceeds the yield shear stress. In this case the mean value of the performance function, $\mu_G = 27.747$ MPa, the standard deviation $\sigma_G = 35.77$ MPa, and the probability that the performance function $G$ is less than zero is 20.2%, $P[G<0] = 20.2\%$.

### Results of Reliability Based Analysis

For various values of the standard deviation $\mu$, the probabilistic FEA model can not only predict the mean value of the performance function $\mu_G$, and the standard deviation of the performance function $\sigma_G$, but also the probability that the performance function is less than zero $P[G < 0]$. Table 1 shows the mean, the standard deviation of performance function $G$ and probabilities of failure for various values of the mean thickness value $\mu_t$. Figure 7 shows a plot of the probability that the performance function is less than zero versus the mean thickness value $\mu_t$. One may observe that for values of $\mu_t = 0.9633$, $0.9281$ and $0.8995$ mm, the probability of failure is 5%, 10% and 15% respectively. This graph can be used as a design guide to select the required mean thickness value $\mu_t$ to achieve the desired reliability level. For example, if the desired reliability level is 95% the minimum mean thickness value $\mu_t$ should be 0.9633 mm.

### Results Of Robust Analysis

The robust design optimization approach not only shifts the performance mean to the target value but also reduces a product’s performance variability, achieving the desired sigma level robustness on the key product performance characteristics with respect to the quantified variation.

In this study a typical formulation of an n-sigma level robust design approach can be stated as:

Find the value of the minimum mean thickness value $\mu_t$ in order to achieve positive values of the expression

$$\mu_G - n \sigma_G > 0$$
Figure 8 shows a sensitivity plot of six curves for 1-σ through 6-σ versus the mean thickness value $\mu_t$. For a 3-σ quality level, the mean thickness value $\mu_t$ should be greater than $\mu_t = 1.0536$ mm. For a 6-σ quality level, the mean thickness value $\mu_t$ should be greater than $\mu_t = 1.2558$ mm.

**CONCLUSIONS**

The example presented demonstrates the advantage of using an automated probabilistic design process that enables engineers to identify better designs that meet the performance objectives and are less sensitive to manufacturing variations.

For a given sigma quality level (i.e. six-sigma) the mean thickness value $\mu_t$ can be determined using the design process described. The results summary is also shown in Table 2.

For a given reliability goal (i.e. 95%) the mean thickness value $\mu_t$ can be determined using the design process described. The results summary is shown in Table 2.

By incorporating the physical scatter into the model, the risk of failing legal or consumer tests can be minimized.

<table>
<thead>
<tr>
<th>$\mu_t$ (mm)</th>
<th>$\mu_G$ (Mpa)</th>
<th>$\sigma_G$ (Mpa)</th>
<th>P[$G&lt;0$] (%)</th>
<th>Min G (Mpa)</th>
<th>Max G (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean t Value</td>
<td>Mean G Value</td>
<td>Standard Deviation of G</td>
<td>Probability that $G &lt; 0$</td>
<td>Minimum G Value</td>
<td>Maximum G Value</td>
</tr>
<tr>
<td>0.85</td>
<td>16.118</td>
<td>38.092</td>
<td>30.78</td>
<td>-241.1</td>
<td>125.3</td>
</tr>
<tr>
<td>0.88</td>
<td>27.747</td>
<td>35.774</td>
<td>20.23</td>
<td>-195.0</td>
<td>126.5</td>
</tr>
<tr>
<td>0.90</td>
<td>34.891</td>
<td>34.301</td>
<td>14.83</td>
<td>-146.5</td>
<td>132.9</td>
</tr>
<tr>
<td>0.95</td>
<td>50.919</td>
<td>30.855</td>
<td>6.25</td>
<td>-115.5</td>
<td>132.6</td>
</tr>
<tr>
<td>0.98</td>
<td>59.384</td>
<td>29.200</td>
<td>3.427</td>
<td>-83.97</td>
<td>140.0</td>
</tr>
<tr>
<td>1.00</td>
<td>64.676</td>
<td>27.901</td>
<td>2.31</td>
<td>-118.5</td>
<td>139.0</td>
</tr>
<tr>
<td>1.10</td>
<td>86.906</td>
<td>23.477</td>
<td>0.180</td>
<td>-50.56</td>
<td>149.6</td>
</tr>
<tr>
<td>1.20</td>
<td>104.03</td>
<td>19.862</td>
<td>0.037</td>
<td>-152.1</td>
<td>156.4</td>
</tr>
<tr>
<td>1.40</td>
<td>128.25</td>
<td>14.860</td>
<td>0.00</td>
<td>50.54</td>
<td>168.7</td>
</tr>
<tr>
<td>1.60</td>
<td>144.25</td>
<td>11.525</td>
<td>0.00</td>
<td>77.88</td>
<td>176.1</td>
</tr>
<tr>
<td>1.80</td>
<td>155.40</td>
<td>9.209</td>
<td>0.00</td>
<td>100.00</td>
<td>181.24</td>
</tr>
<tr>
<td>2.00</td>
<td>163.47</td>
<td>7.5757</td>
<td>0.00</td>
<td>117.2</td>
<td>182.8</td>
</tr>
</tbody>
</table>

**Table 1**: Mean, Standard Deviation of Performance Function $G$ and Probabilities of Failure for various values of the Mean thickness value $\mu_t$. 
### ACKNOWLEDGMENTS

This research effort was partially funded by the Department of Energy (DOE) Office of Advanced Transportation Technologies. The authors would like to express their appreciation to Robert Kost, Vehicle System Team Leader of DOE, Terry Penney, Technology Manager of NREL, Keith Wipke Digital Functional Vehicle Program Manager of NREL, Dan Arbitter, Eleni Beyko, N. Purushothaman and Qutub Khaja of Ford Motor Company for their support to this project.

### REFERENCES

1. ANSYS Inc: Probabilistic Design Techniques

### CONTACTS

Dr. Andreas Vlahinos is the Principal of Advanced Engineering Solutions, LLC, your virtual resource for rapid new product development and professor adjunct at CU-Boulder. He received his Ph.D. in Engineering Science and Mechanics from Georgia Institute of Technology. He has been Professor of Structural Engineering at the University of Colorado teaching courses in Structural Mechanics and in Computer Aided

---

<table>
<thead>
<tr>
<th>Quality / Reliability</th>
<th>Mean Thickness, $\mu_t$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-σ Quality Level</td>
<td>0.8987</td>
</tr>
<tr>
<td>2-σ Quality Level</td>
<td>0.9775</td>
</tr>
<tr>
<td>3-σ Quality Level</td>
<td>1.0536</td>
</tr>
<tr>
<td>4-σ Quality Level</td>
<td>1.1222</td>
</tr>
<tr>
<td>5-σ Quality Level</td>
<td>1.1866</td>
</tr>
<tr>
<td>6-σ Quality Level</td>
<td>1.2558</td>
</tr>
<tr>
<td>85% Reliability Level</td>
<td>0.8995</td>
</tr>
<tr>
<td>90% Reliability Level</td>
<td>0.9281</td>
</tr>
<tr>
<td>95% Reliability Level</td>
<td>0.9633</td>
</tr>
</tbody>
</table>

*Table 2: Mean Thickness $\mu_t$ for various quality level*
Structural Engineering. He has received the Professor of the Year Award several times. He has over 80 publications in areas of structural stability, vibrations, structural dynamics, and design optimization. He has received the R&D 100 award and holds several patents. He has been instrumental in rapid product development through the implementation of Six Sigma and Computer Aided Concurrent Engineering for several Government agencies such as NASA, NREL and DOE and industry partners such as IBM, Coors, Lockheed Martin, Alcoa, Allison Engine Comp., Solar Turbines, Ball, Futech, American Standard, TDM, PTC, MDI, FORD etc.

Dr. Vlahinos can be reached at (303) 814-0455 or his E-mail: andreas@aes.nu

---

Dr. Subhash G. Kelkar is Staff Technical Specialist, Durability CAE at Ford Motor Company. He has published numerous internal and external technical publications. He received his Ph.D. in Mechanical Engineering from the University of Missouri-Rolla. He is currently the Chairman of the Advanced Body Design & Engineering track of the SAE International Body Engineering Conference. His telephone number is: (313)-323-1069; his e-mail address is: skelkar1@ford.com